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# Holistic Electron Theory

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*Abstract*

A relativistic, non-quantum theory of electrons is constructed in which the electron is not considered to be composed of any type of distribution of charge. The electron's structurelessness is defined by several assumptions which, together with Maxwell's equations outside the electron, yield the general fields produced by such an arbitrarily moving *holistic* electron. Several equations of motion for the holistic electron are found to result from the formulation, the Lorentz–Dirac equation being among them. The formulation, by its very nature, avoids the problems of the electron self-energy and the need for normalization.

## 1. *Introduction*

Historically, the electron was considered as a very small sphere characterized by a suitable distribution of charge. Assuming that Maxwell's equations held *everywhere*, and taking into account the retarded self-interaction of such a charge, Lorentz (1952) was even able to derive the 'electron's' equations of motion. His description accounted for the radiation emitted by the charge and also the self-damping force thereby produced. This approach has since been considered by others and may be said to have culminated in the work of Dirac (1938) where, again, the equations of motion of a point electron are derived in covariant form—containing Lorentz's results as a special case. To be sure, points of view other than those of Lorentz and Dirac have been considered,† but it is fair to say that essentially all attempts at dealing with the (non-quantum) electron involve the assumption that the electron is characterized by some distribution of charge or other. (In recent years the distribution is usually taken as that of the Dirac-delta function.) This type of description involves the double assumption that (i) it is meaningful to speak of the electron's field *every-*

† See, for example, F. Rohrlich, *Classical Charged Particles*, Addison-Wesley, Reading, Mass., 1965, for a discussion of various approaches taken and an extensive set of references.

*where* (even though it may be infinite at some event) and (ii) the electron can be considered to be composed of a still finer 'electrical dust' whose effects are also described by the same equations—Maxwell's equations—as are electrons themselves. This latter assumption is, in fact, responsible for the still present difficulty of the self-energy problem and its companion, the renormalization problem, in the non-quantum electron problem.† Further, an omission common to most of the cited work is that the formalisms are never instructed that they are dealing with an electron rather than with just a small cluster of charges.

In actual fact, of course, the *most* that we know is that Maxwell's equations hold everywhere *outside* the electron. We cannot speak of the field within an electron as there is no suitable probe for its measurement and we have no equations that, to our knowledge, are adequate in such a domain. This statement is both well known and ignored.

Accordingly, in the present work, we view the electron *holistically*. That is, we utilize Maxwell's equations only *outside* the electron, and we do not presume that the electron is composed of any finer substance. Its structure and size will be left unspecified, but we shall make several assumptions concerning its structurelessness which comprise, then, our definition of the electron. On this basis, we shall first discuss electrodynamics where we recover the customary Lienard–Wiechert potentials as the general field produced by the holistic electron. Secondly, we consider the equations of motion of the holistic electron and find, as a consequence of our assumptions, that several equations are possible, the Lorentz–Dirac equation being among them.

Finally, we note that, even though perhaps nothing new is gained from holistic considerations, it is at least esthetically pleasing (to this author) that the important consequences of customary electrodynamics can be derived without making indefensible assumptions concerning the nature of the electron. And further, that the considerations are not tainted by the spurious issues of the self-energy and mass renormalization of the electron

## 2. *Holistic Electrodynamics*

In this section our goal will be to derive the **E** and **H** fields produced by an arbitrarily moving *holistic* electron.

We begin with Maxwell's equations in the form

$$\nabla \times \mathbf{E} = -\dot{\mathbf{H}}/c, \qquad \nabla \times \mathbf{H} = \dot{\mathbf{E}}/c, \qquad \nabla . \mathbf{H} = 0, \qquad \nabla . \mathbf{E} = 0 \qquad (2.1)$$

which are assumed to hold in any Lorentz frame everywhere *outside* the electron.

† Op. cit., Chapters 6 and 7. Also, an interesting treatment of this problem is to be found by M. H. L. Pryce, *Proceedings of the Royal Society*, A168, 389 (1938), who reduces the self-energy to zero, but does not treat the electron holistically. Finally, a recent work where self-energy problems are avoided is found in S. L. Schwebel, *International Journal of Theoretical Physics*, Vol. 6, No. 1, p. 61.

We see that, if $\Sigma$ denotes a stationary closed surface which may, or may not, contain the electron, then the last equation above implies that $\int_\Sigma \mathbf{E}.\mathbf{n}d\sigma$ is independent of the shape of $\Sigma$ if $\Sigma$ contains the electron (this is also true, of course, if $\Sigma$ contains no electron). The second equation above then implies that the value of $\int_\Sigma \mathbf{E}.\mathbf{n}d\sigma$ is time-independent as long as $\Sigma$ either does or does not contain the electron. We denote the constant, $\int_\Sigma \mathbf{E}.\mathbf{n}d\sigma/4\pi$, where $\Sigma$ contains one electron, as the charge, $-e$ ($e > 0$), of the electron.

Continuing on, as in the customary electrodynamics, we assume the existence of the field tensor, $F^{\mu\nu}$, given by

$$F^{\mu\nu} \sim \begin{pmatrix} 0 & H_3 & -H_2 & E_1 \\ -H_3 & 0 & H_1 & E_2 \\ H_2 & -H_1 & 0 & E_3 \\ -E_1 & -E_2 & -E_3 & 0 \end{pmatrix} \tag{2.2}$$

where Greek indices go from 1 to 4, and the metric is $g_{\mu\nu} = \text{diag}(1,1,1,-1)$, and we take

$$c^2\,d\tau^2 = -g_{\mu\nu}\,dx^\mu\,dx^\nu = c^2\,dt^2 - ds^2$$

In covariant form, we have Maxwell's equations as

$$F^{\mu\nu}_{,\nu} = 0, \qquad F_{\mu\nu,\lambda} + F_{\nu\lambda,\mu} + F_{\lambda\mu,\nu} = 0 \tag{2.3}$$

For further progress, we now make two (of three) assumptions that we take as defining the electron:

(A) The electron is 'structureless' in the sense that, the 4-vector potential field, $A^\mu$, it produces has a dependence on the electron's kinematical history which has the same *form* whether the electron is accelerating or not.

(B) The fields produced by the electron are propagated with the speed of light.

We are now able to construct an argument giving the field produced by an arbitrarily moving holistic electron—as follows:

First, consider a single stationary electron, whose field is then spherically symmetric. Letting $\varphi(r)$ denote the electron's scalar potential, we have that $\nabla^2\varphi = 0$ everywhere outside the electron, which, with the conditions that $\int_\Sigma \nabla\varphi.\mathbf{n}d\sigma = -4\pi e$, and $\varphi \to 0$ as $r \to \infty$, uniquely gives the potential everywhere outside the electron as $\varphi(r) = e/r$.

Denoting the rest frame of this stationary electron with a naught superscript, we may now write the 4-vector potential in this frame as

$$A^{\mu(0)} = \left[\frac{eV^\mu}{c\rho}\right]^{(0)} \tag{2.4}$$

where $V^\mu = dx^\mu/d\tau$ is the 4-velocity of the electron, and $\rho$—for the moment—signifies $r$.

There are many ways that the invariant, $\rho$, can be defined compatibly with its value of $r$ in the electron's (permanent) rest system. Considering assumption (B) we take $\rho$ to be the distance, in the rest frame, between the charge's retarded location and the location of the field point.

We now have, for a *uniformly* moving electron, the 4-potential in any Lorentz frame as

$$A^{\mu} = \frac{eV^{\mu}}{c\rho} \tag{2.5}$$

Finally, using assumption (A), we can now take the expression

$$A^{\mu} = \frac{eV^{\mu}_{\text{ret.}}}{c\rho_{\text{ret.}}} \tag{2.6}$$

as the 4-vector potential in any Lorentz frame, as produced by an *arbitrarily* moving electron.

Thus, we recover the customary Lienard–Wiechert expression for the potentials and thereby the customary expression for the fields of an arbitrarily moving holistic electron.

## 3. *Equations of Motion*

Here, we construct the equations of motion for the holistic electron. Toward this end, we make a final assumption defining the electron:†

(C) If an external force, $\mathbf{F}_{\text{ext}}$, acts on an electron, thus transporting it from some initial velocity and acceleration $(\mathbf{v}, \mathbf{a})$ back to the same $\mathbf{v}$ and $\mathbf{a}$ via any 'velocity path', then the energy and momentum contributed by this force must equal that radiated by the electron during this process.

Loosely speaking, this assumption reflects that the electron is structureless in the sense that it cannot absorb energy or momentum. In relation to this we also consider the *observed* electron rest mass, $M_0$, to be strictly a constant.

Next, we note that the rate of energy-momentum loss, $dp^{\mu}_{\text{rad}}/d\tau$, by the holistic electron has the same form as one finds for a point charge in the conventional treatment‡ since the result only depends on the Lienard–Wiechert potentials. It is given by

$$\frac{dp^{\mu}_{\text{rad}}}{d\tau} = -\frac{2}{3}\frac{e^2}{c^5} a^{\lambda} a_{\lambda} V^{\mu} \tag{3.1}$$

where $a^{\sigma} = dV^{\sigma}/d\tau$.

† Here we are referring to a work by the author, *American Journal of Physics*, 35, 949 (1967), where this assumption was originally made. This work, however, treats the electron non-holistically.

‡ See, for instance, F. Rohrlich, *Classical Charged Particles*, p. 111, Addison-Wesley, Reading, Mass., 1965.

Now consider the holistic electron moving under the influence of an external 4-force, $F^{\mu}_{ext}$. Assuming conservation of momentum between electron and field, we have

$$M_0 a^{\mu} = -\frac{2}{3}\frac{e^2}{c^5} a^{\lambda} a_{\lambda} V^{\mu} + D^{\mu} + F^{\mu}_{ext} \tag{3.2}$$

where the (as yet undetermined) 4-vector $D^{\mu}$ is added since the first term on the right may not include all the energy-momentum in the field.

If $F^{\nu}_{ext}$ is such that the electron is taken from some $(\mathbf{v}, \mathbf{a})$ along some path back to the same $(\mathbf{v}, \mathbf{a})$, then assumption (C) implies that†

$$D^{\mu} = \frac{dc^{\mu}}{d\tau} \equiv \dot{c}^{\mu} \tag{3.3}$$

for some 4-vector $c^{\mu}$, which is necessarily of the form, $c^{\mu} = \alpha a^{\mu} + \beta V^{\mu}$, where $\alpha$ and $\beta$ may be functions of kinematical variables.

We then assume that for *any* external force (say an electromagnetic force where $F^{\mu}_{ext} = (e/c) F^{\mu\nu}_{ext} V_{\nu}$) we have

$$M_0 a^{\mu} = -\frac{2}{3}\frac{e^2}{c^5} a^{\lambda} a_{\lambda} V^{\mu} + D^{\mu} + \frac{e}{c} F^{\mu\nu}_{ext} V_{\nu} \tag{3.4}$$

Now, $D^{\mu} = \dot{\alpha} a^{\mu} + \alpha \dot{a}^{\mu} + \beta a^{\mu} + \dot{\beta} V^{\mu}$. Contracting (3.4) with $V^{\mu}$ then yields

$$\frac{2}{3}\frac{e^2}{c^3} a^{\lambda} a_{\lambda} = \dot{\beta} c^2 + \alpha a^{\sigma} a_{\sigma} \tag{3.5}$$

where we have used the relations, $F^{\mu\nu}_{ext} V_{\mu} V_{\nu} = 0$, and $\dot{a}^{\mu} V_{\mu} = -a_{\sigma} a^{\sigma}$.

We now divide our considerations into three cases:

*Case (I).* $\beta = 0$ for all $\tau$.

Then $\dot{\beta} = 0$, which implies that $\alpha = 2e^2/3c^3$. Further then, $C^{\mu} = 2e^2 a^{\mu}/3c^3$, implying that $D^{\mu} = 2e^2 \dot{a}^{\mu}/3c^3$. Therefore, we have in this case,

$$M_0 a^{\mu} = -\frac{2}{3}\frac{e^2}{c^5} a^{\lambda} a_{\lambda} V^{\mu} + \frac{2}{3}\frac{e^2}{c^3} \dot{a}^{\mu} + \frac{e}{c} F^{\mu\nu}_{ext} V_{\nu} \tag{3.6}$$

which is the Lorentz–Dirac equation.

*Case (II).* $\beta = \text{const.} \neq 0$.

Then $\dot{\beta} = 0$ again, and $\alpha = 2e^2/3c^3$. This implies that $D^{\mu} = 2e^2 \dot{a}^{\mu}/3c^3 + \beta a^{\mu}$, and, therefore, that

$$M_0 a^{\mu} = -\frac{2}{3}\frac{e^2}{c^5} a^{\lambda} a_{\lambda} V^{\mu} + \frac{2}{3}\frac{e^2}{c^3} \dot{a}^{\mu} + \beta a^{\mu} + F^{\mu\nu}_{ext} \tag{3.7}$$

In this case, the electron moves precisely as a particle with mass, $\tilde{M}_0 \equiv M_0 - \beta$, where $\beta$ is an undetermined constant.

† See footnote †, p. 182.

*Case (III)*. $\beta \neq$ const.

Then,

$$D^{\mu} = \alpha \dot{a}^{\mu} + (\dot{\alpha} + \beta) a^{\mu} + \left(\frac{2}{3}\frac{e^2}{c^3} - \alpha\right)\frac{1}{c^2} a^{\lambda} a_{\lambda} V^{\mu}$$

and we obtain,

$$M_0 a^{\mu} = \alpha \dot{a}^{\mu} + (\dot{\alpha} + \beta) a^{\mu} - \frac{1}{c^2}\alpha a^{\lambda} a_{\lambda} V^{\mu} + \frac{e}{c} F^{\mu\nu}_{\text{ext}} V_{\nu} \tag{3.8}$$

as the equations of motion, where $\alpha$ and $\beta$ are undetermined. Actually, this relation only depends on the single quantity, $\beta$, if one uses relation (3.5) to eliminate $\alpha$.

Thus, our considerations allow three possible equations of motion. However, the Dirac equation is outstanding as the only one not depending on undetermined parameters.

## *References*

Dirac, P. A. M. (1938). *Proceedings of the Royal Society*, A167, 148.

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